Orbital Energy

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1.1 কক্ষপথের প্রকার বনাম গতির প্রকার — Types of Orbit vs. Type of Motion

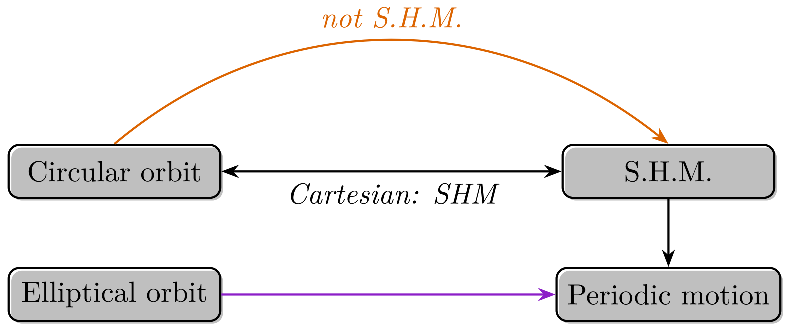
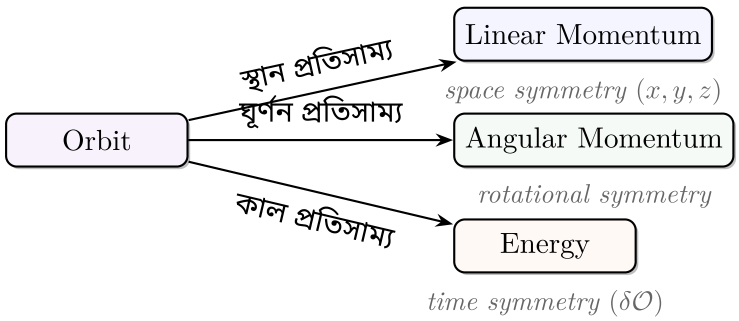


Figure 1: Relationship between orbit type and type of motion. A circular orbit is equivalent to S.H.M. in each Cartesian coordinate (bidirectional arrow). An elliptical orbit gives periodic motion; S.H.M. gives periodic motion but circular motion is *not* S.H.M. in the polar-radial coordinate.

1.2 প্রতিসাম্য থেকে তিনটি সংরক্ষণ সূত্র — Three Conservation Laws from Symmetry

কক্ষীয় শক্তি (**Orbital Energy**) মৌলিক ধারণার সাথে যুক্ত:

$$\text{Law} \rightarrow \text{Conservation} \longleftarrow \text{Symmetry}.$$



A concrete example from kinematics:

$$s = ut + \tfrac{1}{2}at^2.$$

সময় প্রতিসাম্য: (**Time symmetry:**) গতির সমীকরণ $t \to +t$ ও $t \to -t$ তে অপরিবর্তনশীল.

2 কেন্দ্রীয় বল গতি — Central Force Motion

A **central force** (কেন্দ্রীয় বল) satisfies two conditions:

1. Its direction always points *toward* (or away from) a fixed centre.
2. Its magnitude depends only on the *distance* r from the centre.

$$\vec{F} = f(r)\,\hat{r}$$

এখানে $f(r) < 0$ মানে (**attractive**); $f(r) > 0$ মানে (**repulsive**); এবং $|\vec{F}| = f(r)$.

Common functional forms (three important cases):

$$f(r) = \alpha r, \qquad f(r) = \frac{\alpha}{r}, \qquad f(r) = \frac{\beta}{r^2}.$$

These correspond to the isotropic oscillator (r), logarithmic potential ($1/r$), and the Kepler/Coulomb problem ($1/r^2$).

2.1 কেন্দ্রীয় বল কেন? দ্বি-বস্তু সমস্যা — Why Central Force? The Two-Body Problem

The two-body problem reduces as follows:

$$\text{Two-body problem} \xrightarrow{\text{reduced}} \text{One-body problem} \rightarrow \text{Central force problem.}$$

$$\text{Central force problem} \rightarrow \text{Effective potential.}$$

The force depends only on **distance** (one parameter):

$$x \rightarrow \dot{x} \rightarrow \ddot{x} \qquad (v,\ a).$$

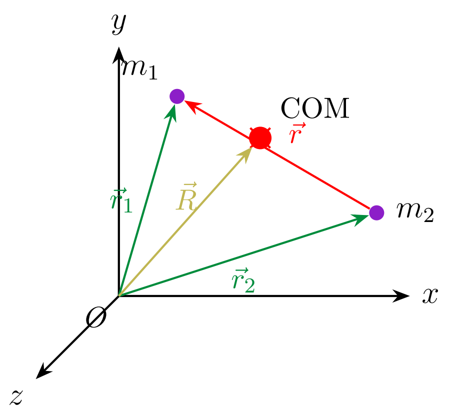
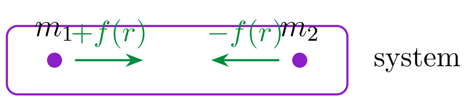


Figure 2: Two-body system. $\vec{r}_1, \vec{r}_2$: position vectors from origin O; $\vec{r} = \vec{r}_1 - \vec{r}_2$: relative displacement; $\vec{R}$: centre-of-mass (COM) vector.

Newton's second law for each body:

$$m_1\,\ddot{\vec{r}}_1 = +f(r)\,\hat{r} \tag{i}$$
$$m_2\,\ddot{\vec{r}}_2 = -f(r)\,\hat{r} \tag{ii}$$



3 এক-বস্তু সমস্যায় রূপান্তর — Reduction to a One-Body Problem

3.1 ভরকেন্দ্র কাঠামো — Centre-of-Mass Frame

Define:

$$\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}, \qquad \vec{r} = \vec{r}_1 - \vec{r}_2.$$

Note: $\vec{R} \to \text{COM}$, $\dot{\vec{R}} \to v_{\text{COM}}$, $\ddot{\vec{R}} \to a_{\text{COM}}$.

Adding (i) and (ii):

$$m_1\ddot{\vec{r}}_1 + m_2\ddot{\vec{r}}_2 = \vec{0} \implies \frac{m_1\ddot{\vec{r}}_1 + m_2\ddot{\vec{r}}_2}{m_1 + m_2} = \vec{0} \implies \ddot{\vec{R}} = 0.$$

So the COM moves freely (constant velocity if any initial velocity exists).

From (i) and (ii):

$$\ddot{\vec{r}}_1 = \frac{f(r)}{m_1}\hat{r}, \qquad \ddot{\vec{r}}_2 = -\frac{f(r)}{m_2}\hat{r}.$$

Since $\vec{r} = \vec{r}_1 - \vec{r}_2$, differentiate twice:

$$\ddot{\vec{r}} = \ddot{\vec{r}}_1 - \ddot{\vec{r}}_2, \qquad \boxed{\ddot{\vec{r}} = \ddot{\vec{r}}_1 - \ddot{\vec{r}}_2}.$$

Substituting:

$$\ddot{\vec{r}}_1 - \ddot{\vec{r}}_2 = \left(\frac{1}{m_1} + \frac{1}{m_2}\right) f(r)\hat{r} = \frac{m_1 + m_2}{m_1 m_2} f(r)\hat{r} = \ddot{\vec{r}}.$$

3.2 হ্রাসকৃত ভর — Reduced Mass

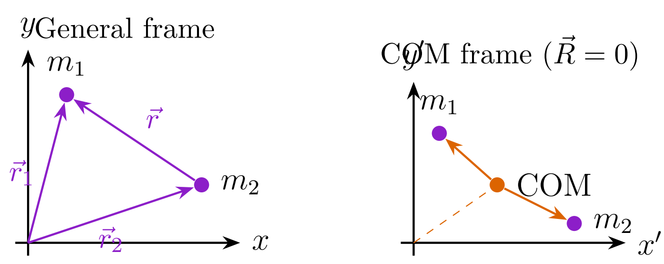


Figure 3: Left: general reference frame. Right: COM frame where the COM is fixed at the origin.

Define the **reduced mass** (হ্রাসকৃত ভর):

$$\mu = \frac{m_1 m_2}{m_1 + m_2}.$$

Then the equation of motion becomes:

$$f(r)\,\hat{r} = \mu\,\ddot{\vec{r}}.$$

Equation of Motion for Any Central Force

$$f(r)\,\hat{r} = \mu\,\ddot{\vec{r}}$$

Here r is the distance between the two objects. Taking the COM as reference frame ($\vec{R} = 0$):

$$\vec{r}_1 = +\frac{m_2}{m_1 + m_2}\vec{r}, \qquad \vec{r}_2 = -\frac{m_1}{m_1 + m_2}\vec{r}, \qquad \boxed{\vec{R} = 0}.$$

4 কৌণিক ভরবেগের সংরক্ষণ — Conservation of Angular Momentum

4.1 কেন্দ্রীয় বলের টর্ক শূন্য — Torque of a Central Force is Zero

The torque about the origin:

$$\vec{r} \times \vec{F} = \vec{r} \times f(r)\hat{r} = f(r)\,\underbrace{(\vec{r} \times \hat{r})}_{=\vec{0}} = \vec{0}.$$

Therefore:

$$\tau_{\text{ext}} = 0 \implies \tau_{\text{on system}} = 0.$$

4.2 $\vec{L}$ সংরক্ষণের প্রমাণ — Proof that $\vec{L}$ is Conserved

With $\vec{p} = \mu\vec{v}$, the angular momentum is $\vec{L} = \vec{r} \times \vec{p}$. Computing:

$$\frac{dL_{\text{total}}}{dt} = \underbrace{\vec{r} \times \vec{F}}_{=\vec{0}} + \underbrace{\vec{v} \times \mu\vec{v}}_{=\vec{0}} = \vec{0} \implies \tau_{\text{total}} = 0.$$

$$\vec{L} = \text{conserved}$$

4.3 ভরকেন্দ্র কাঠামোতে কৌণিক ভরবেগ — Angular Momentum in the COM Frame

With the COM at the origin:

$$\vec{L} = \vec{r}_1 \times m_1\vec{v}_1 + \vec{r}_2 \times m_2\vec{v}_2 = \mu\,\vec{r} \times \vec{v} \equiv \boxed{\mu\,\vec{r} \times \vec{v}}.$$

4.4 $\vec{L}$ সংরক্ষণের ফলাফল — Consequences of Conservation of $\vec{L}$

Since $\vec{L}$ is conserved, it has:

1. **Fixed magnitude.**
2. **Fixed direction.**
3. $\implies$ The motion is **planar** (restricted to a single plane). *[2D!]*

5 কেপলারের সূত্র — Kepler's Laws

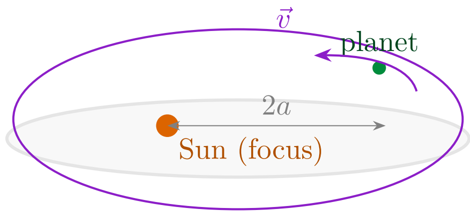


Figure 4: Kepler orbit: a planet moves in an elliptical, planar orbit with the Sun at one focus.

১ম সূত্র — 1st Law. The orbit is an ellipse $\to$ planar.

২য় সূত্র — 2nd Law. The areal velocity is constant.

২য় সূত্রের ব্যুৎপত্তি — Derivation of 2nd Law

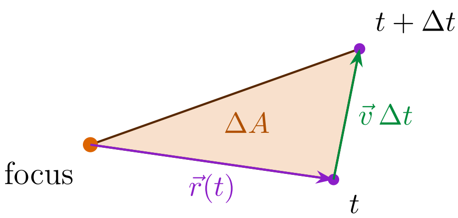


Figure 5: Area ΔA swept by $\vec{r}$ in time Δt. For small Δt, $\Delta A \approx \frac{1}{2}|\vec{r} \times \vec{v}|\,\Delta t$.

$$\Delta A = \tfrac{1}{2}\,|\vec{r} \times \vec{v}|\,\Delta t = \frac{|\vec{L}|}{2\mu}\,\Delta t \implies \frac{\Delta A}{\Delta t} = \frac{L}{2\mu} = \text{const.}$$

(constant because $L = |\vec{L}| = \text{const.}$)

Summary:

$$\left.\begin{array}{l} \checkmark\ \text{1st Law (ellipse} \to \text{planar)} \\[1em] \checkmark\ \text{2nd Law (areal velocity const.)} \end{array}\right\} \text{both follow from } \vec{L} = \text{conserved.}$$

৩য় সূত্র — 3rd Law. Requires Newton's gravitational law, $F \propto 1/r^2$ (holds for point particles):

$$\boxed{F \propto \frac{1}{r^2}} \implies \boxed{T^2 \propto a^3}.$$

For two extended bodies, replace mass m with reduced mass μ.

6 বৃত্তাকার কক্ষপথ — Circular Orbit: Simplest Case

For a circular orbit of radius R:

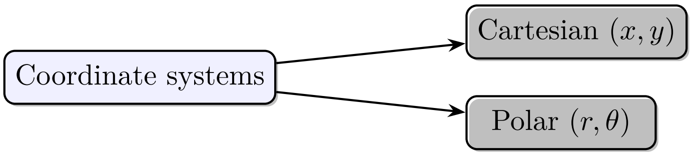
$$T = \frac{2\pi R}{v}, \qquad F = \frac{mv^2}{R}, \qquad F \propto \frac{1}{R^2}.$$

Both $\vec{L}$ and E are conserved.

7 পোলার স্থানাঙ্কে গতিবিদ্যা — Kinematics in Polar Coordinates

7.1 স্থানাঙ্ক ব্যবস্থা — Coordinate Systems

Dynamics/Kinematics in 2D:



7.2 ঘূর্ণন গতির জন্য কার্তেসীয় কেন ব্যর্থ — Why Cartesian Fails for Rotating Motion

For **linear (Cartesian) motion**: $\vec{r} = x\hat{\imath}$, and since $\hat{\imath} = \text{const.}$,

$$\frac{d\vec{r}}{dt} = \dot{x}\hat{\imath} + \cancelto{0}{\frac{d\hat{\imath}}{dt}} x = \dot{x}\hat{\imath}.$$

But for **rotating motion**, $\frac{d\hat{r}}{dt} \neq 0$: this is *not* linear motion.

কার্তেসীয় রূপ: কেন্দ্রীয় বলের সমীকরণ — Cartesian form (messy!)

In Cartesian coordinates $\vec{r} = x\hat{\imath} + y\hat{\jmath}$, $\ddot{\vec{r}} = \ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath}$, so $\mu\ddot{\vec{r}} = f(r)\hat{r}$ gives:

$$\mu\ddot{x} = f\left(\sqrt{x^2+y^2}\right)\frac{x}{\sqrt{x^2+y^2}}, \qquad \mu\ddot{y} = f\left(\sqrt{x^2+y^2}\right)\frac{y}{\sqrt{x^2+y^2}}.$$

These coupled ODEs are messy. **Polar coordinates are far more natural.**

7.3 পোলার একক ভেক্টর $\hat{r}$ ও $\hat{\theta}$ — Polar Unit Vectors $\hat{r}$ and $\hat{\theta}$

In Cartesian components:

$$\hat{r} = \cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}, \qquad \hat{\theta} = -\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath}.$$

Magnitudes: $|\hat{r}| = |\hat{\theta}| = 1$, and $(r, \theta) \to (\hat{r}, \hat{\theta})$ with $\hat{r} \perp \hat{\theta}$.

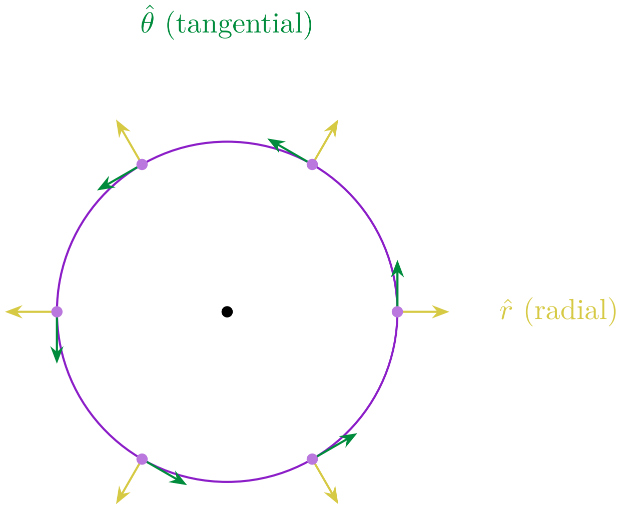


Figure 6: Polar unit vectors $\hat{r}$ (radial, outward) and $\hat{\theta}$ (tangential/azimuthal) at six points on a circle. Both vectors rotate with the particle; $\hat{r} \perp \hat{\theta}$. For a circle $|\vec{r}| = \text{const.}$ but $\hat{r} \neq \text{const.}$

একক ভেক্টরের অন্তরজ — Derivatives of the unit vectors

$$\frac{d\hat{r}}{d\theta} = \hat{\theta}, \qquad \frac{d\hat{\theta}}{d\theta} = -\hat{r}.$$

In time (using $\dot{\theta} \equiv \omega$):

$$\frac{d\hat{r}}{dt} = \dot{\theta}\,\hat{\theta}, \qquad \frac{d\hat{\theta}}{dt} = -\dot{\theta}\,\hat{r}.$$

7.4 পোলার স্থানাঙ্কে বেগ — Velocity in Polar Coordinates

Since $\vec{r} = r\hat{r}$:

$$\dot{\vec{r}} = \dot{r}\hat{r} + r\,\frac{d\hat{r}}{dt} = \dot{r}\hat{r} + r\dot{\theta}\,\hat{\theta}.$$

$$\boxed{\vec{v} = \dot{r}\,\hat{r} + r\dot{\theta}\,\hat{\theta} = v_r\,\hat{r} + v_t\,\hat{\theta}, \qquad v_r = \dot{r}, \quad v_t = r\dot{\theta} = r\omega.}$$

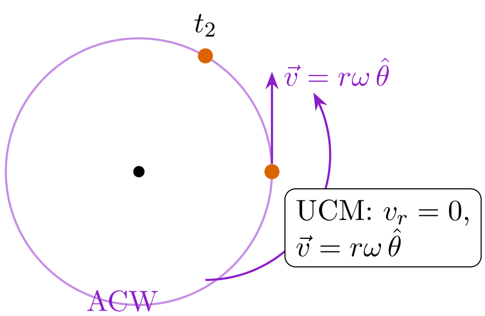


Figure 7: Velocity in circular motion. For uniform circular motion $v_r = 0$ and $\vec{v} = r\omega\,\hat{\theta}$ always tangent to the circle.

7.5 পোলার স্থানাঙ্কে ত্বরণ — Acceleration in Polar Coordinates

Differentiating $\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$:

$$\vec{a} = \ddot{\vec{r}} = \underbrace{\left(\ddot{r} - r\dot{\theta}^2\right)}_{a_{\text{rad}}}\hat{r} + \underbrace{\left(2\dot{r}\dot{\theta} + r\ddot{\theta}\right)}_{a_{\tan}}\hat{\theta}.$$

বৃত্তাকার গতি — Circular motion ($|\vec{r}| =$ **const.**, $\dot{r} = 0$, $\ddot{r} = 0$)

$$\vec{a} = -r\dot{\theta}^2\,\hat{r} + r\ddot{\theta}\,\hat{\theta} = -r\omega^2\,\hat{r} + r\alpha\,\hat{\theta}.$$

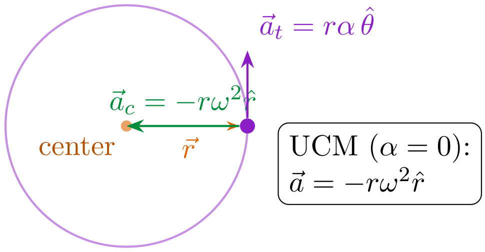


Figure 8: Centripetal acceleration $\vec{a}_c = -r\omega^2\hat{r}$ (towards centre) and tangential acceleration $\vec{a}_t = r\alpha\,\hat{\theta}$ in circular motion. For **Uniform Circular Motion (UCM)** $\alpha = 0$, so $\vec{a} = -r\omega^2\hat{r}$.

8 পোলার স্থানাঙ্কে গতির সমীকরণ — Equations of Motion in Polar Coordinates

Applying $f(r)\hat{r} = \mu\vec{a}$ (no tangential force in a central force):

Radial component:

$$\mu\left(\ddot{r} - r\dot{\theta}^2\right) = f(r). \tag{R}$$

Tangential component:

$$\boxed{\mu\left(2\dot{r}\dot{\theta} + r\ddot{\theta}\right) = 0}$$

Multiplying by r:

$$2r\dot{r}\dot{\theta} + r^2\ddot{\theta} = 0 \implies \frac{d}{dt}\left(r^2\dot{\theta}\right) = 0 \implies r^2\dot{\theta} = \text{const.}$$

8.1 কৌণিক ভরবেগের পুনঃপ্রাপ্তি — Angular Momentum Recovered

আমরা দেখি $\vec{L} = \mu\vec{r} \times \vec{v}$ থেকে:

$$\vec{L} = \mu\vec{r} \times \vec{v} = \mu r\hat{r} \times \left(\dot{r}\hat{r} + r\dot{\theta}\hat{\theta}\right) = \mu r^2\dot{\theta}\,\underbrace{\hat{r} \times \hat{\theta}}_{=\hat{\eta}}.$$

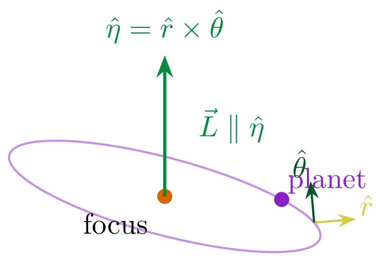


Figure 9: Orbital plane and its normal $\hat{\eta} = \hat{r} \times \hat{\theta}$. Angular momentum $\vec{L} = \mu r^2 \dot{\theta}\,\hat{\eta}$ is perpendicular to the plane.

Here $\hat{\eta} = \hat{r} \times \hat{\theta}$ is the unit normal to the orbital plane (**কক্ষতলের একক লম্ব**).

$$|\vec{L}| = \mu r^2 \dot{\theta} = \ell \implies \boxed{r^2 \dot{\theta} = \frac{\ell}{\mu}}, \qquad \dot{\theta} = \frac{\ell}{\mu r^2}.$$

8.2 রেডিয়াল গতির সমীকরণ — Radial Equation of Motion

Substitute $\dot{\theta} = \ell/(\mu r^2)$ into (R):

$$\mu \ddot{r} - \mu r \left(\frac{\ell}{\mu r^2} \right)^2 = f(r) \implies \mu \ddot{r} - \frac{\mu \ell^2}{\mu^2 r^3} = f(r).$$

Radial Equation of Motion

$$\mu \ddot{r} - \underbrace{\frac{\ell^2}{\mu r^3}}_{\text{extra (centrifugal) term}} = f(r)$$

This is a **1D equation** for a particle moving radially, with an effective additional (centrifugal) force $+\ell^2/(\mu r^3)$.

Verification: since $\ell = \mu r^2 \dot{\theta}$,

$$\frac{\ell^2}{\mu r^3} = \frac{\mu^2 r^4 \dot{\theta}^2}{\mu r^3} = \mu r \dot{\theta}^2 = \mu r \omega^2 = \boxed{\mu r \omega^2},$$

which is the familiar centrifugal acceleration term.

Force analysis: radial $\to$ equation (R); tangential $\to$ angular momentum conservation.

9 শক্তি ও কার্যকর বিভব — Energy and the Effective Potential

9.1 কেন্দ্রীয় বলের বিভব শক্তি — Potential Energy for a Central Force

A central force is **conservative** (সংরক্ষণশীল):

$$\vec{F} = f(r)\hat{r} \to \text{conservative force}, \qquad \vec{F} = -\nabla U \implies \vec{F}_c = -\frac{dU}{dr}.$$

Therefore:

$$dU = -\vec{F}_c \cdot d\vec{r}, \qquad U(\vec{r}) = -\int_{\vec{r}_0}^{\vec{r}} \vec{F} \cdot d\vec{r} = -\int_{r_0}^{r} f(r')\,\underbrace{\hat{r} \cdot d\vec{r}}_{=dr'}\,dr' = U(r).$$

9.2 মোট শক্তি — Total Energy

$$\text{K.E.} + \text{P.E.} = E \quad \text{(conserved)}.$$
$$\frac{1}{2}\mu v^2 + U(r) = E.$$

In polar coordinates, $\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$, so:

$$\boxed{v^2 = \dot{r}^2 + (r\dot{\theta})^2}$$

$$\frac{1}{2}\mu\left(\dot{r}^2 + r^2\dot{\theta}^2\right) + U(r) = E.$$

9.3 কার্যকর বিভব — Effective Potential

Substitute $\dot{\theta} = \ell/(\mu r^2)$:

$$\frac{1}{2}\mu\dot{r}^2 + \frac{1}{2}\mu r^2\left(\frac{\ell}{\mu r^2}\right)^2 + U(r) = E$$
$$\frac{1}{2}\mu\dot{r}^2 + \frac{1}{2} \cdot \frac{\mu\,\ell^2}{\mu^2 r^2} + U(r) = E.$$

$$\frac{1}{2}\mu\dot{r}^2 + \underbrace{\frac{\ell^2}{2\mu r^2} + U(r)}_{U_{\text{eff}}(r)} = E$$

$$\boxed{U_{\text{eff}}(r) = U(r) + \frac{\ell^2}{2\mu r^2}}$$

এটি দ্বিমাত্রিক সমস্যাকে (1D radial motion) হ্রাস করে:

$$\frac{1}{2}\mu\dot{r}^2 + U_{\text{eff}}(r) = E \quad \longleftrightarrow \quad \frac{1}{2}m\dot{x}^2 + U(x) = E.$$

কার্যকর বিভব থেকে বল — Force from the effective potential

$$-\frac{d}{dr}U_{\text{eff}}(r) = \underbrace{-\frac{dU}{dr}}_{=f(r)} - \frac{d}{dr}\left(\frac{\ell^2}{2\mu r^2}\right) = f(r) + \frac{\ell^2}{\mu r^3}.$$

$$\mu\ddot{r} = -\frac{dU}{dr} + \frac{\ell^2}{\mu r^3}.$$

$$\boxed{\mu\dot{r}\ddot{r} + \left(\frac{dU}{dr} - \frac{\ell^2}{\mu r^3}\right)\dot{r} = 0}$$

10 রেডিয়াল গতি ও কার্যকর বিভবের লেখচিত্র — Radial Motion and the Effective Potential

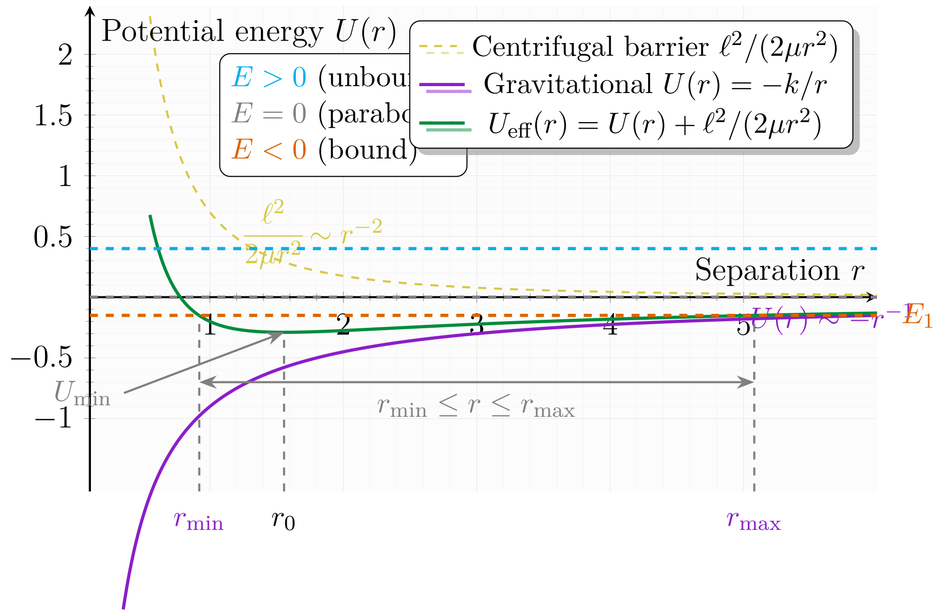


Figure 10: Effective potential $U_{\text{eff}}(r) = U(r) + \ell^2/(2\mu r^2)$ for a gravitational $U(r) = -k/r$ potential. The centrifugal barrier (yellow, $\sim r^{-2}$) prevents the particle from reaching $r = 0$. At $E = U_{\text{min}}$: circular orbit at r_0. For $U_{\text{min}} < E < 0$: bounded radial motion between r_{min} and r_{max}. For $E \geq 0$: unbounded orbit (parabolic/hyperbolic).

$$U_{\text{eff}}(r) = U(r) + \frac{\ell^2}{2\mu r^2}.$$

$$E_{\text{min}} \rightarrow r_0 \rightarrow \text{orbit} \rightarrow \text{circular orbit}.$$

10.1 বৃত্তাকার কক্ষপথের ব্যাসার্ধ r_0 — Circular Orbit Radius r_0

At the minimum of U_{eff}:

$$\frac{d}{dr}U_{\text{eff}}(r)\bigg|_{r=r_0} = 0.$$

For gravity, $U(r) = -k/r$ (where $k = Gm_1m_2$):

$$\frac{d}{dr}\left(-\frac{k}{r}\right) + \frac{d}{dr}\left(\frac{\ell^2}{2\mu r^2}\right) = 0 \implies \frac{k}{r^2} - \frac{\ell^2}{\mu r^3} = 0 \implies k = \frac{\ell^2}{\mu r}.$$

$$r_0 = \frac{\ell^2}{\mu k}$$

At r_0, the tangential kinetic energy:

$$\frac{1}{2}\mu v_0^2 = \frac{\ell^2}{2\mu r^2} = \frac{1}{2}\mu r^2\dot{\theta}^2 \quad \text{(tangential)}.$$

10.2 গ্রহগতির বর্তন বিন্দু — Turning Points of Planetary Motion

The radial motion has **turning points** $r_{\min}$ and $r_{\max}$ where $\dot{r} = 0$ (momentarily at rest in the radial direction).

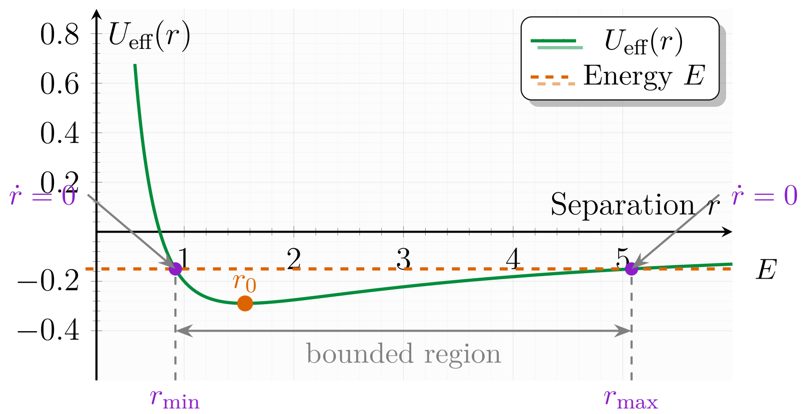


Figure 11: Turning points $r_{\min}$ and $r_{\max}$ where $\dot{r} = 0$ (the radial velocity vanishes, so the motion is "momentarily radial at rest").

11 কক্ষপথের প্রকারভেদ — Types of Orbits

11.1 শক্তি-কক্ষপথ সঙ্গতি — Energy-Orbit Correspondence

For $E_{\min} < E < 0$, the radial motion is bounded:

- **Radial motion** is confined between $r_{\max}$ and $r_{\min}$.
- **Tangential motion** is always in one direction.

At the turning points ($\dot{r} = 0$), angular momentum conservation gives:

$$L_{\min} = L_{\max} \implies \mu v_1 r_{\min} = \mu v_2 r_{\max} \implies \boxed{\frac{v_1}{v_2} = \frac{r_{\max}}{r_{\min}}}.$$

At $r_{\max}$: $r\uparrow$, $v\downarrow$. At $r_{\min}$: $r\downarrow$, $v\uparrow$. ($L = \text{const.}$)

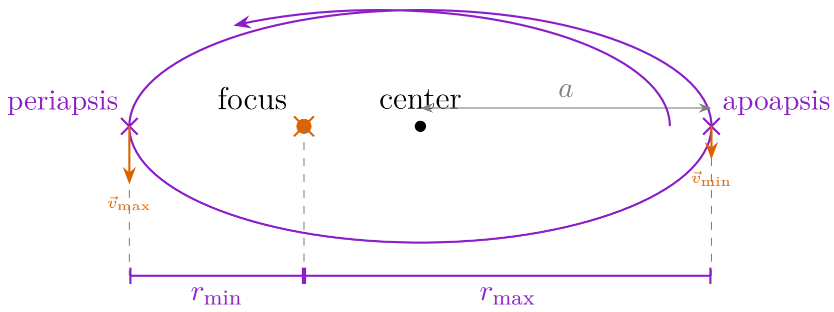


Figure 12: Elliptical orbit showing semi-major axis a, periapsis ($r_{\min}$) and apoapsis ($r_{\max}$). At both turning points $\dot{r} = 0$ (marked $\times$). The primary body is at one focus ($\oplus$). Velocity arrows indicate $\vec{v}_{\max} > \vec{v}_{\min}$ (Kepler's 2nd law).

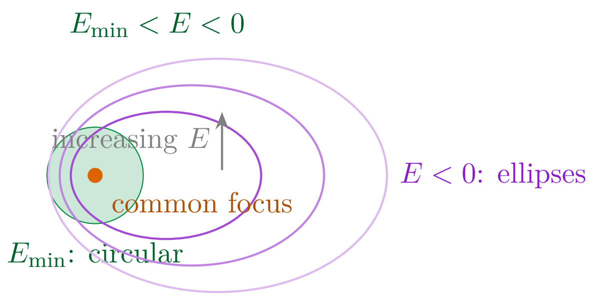


Figure 13: Family of orbits sharing a common focus. Innermost circle at $E = E_{\min}$ (radius r_0); larger ellipses for $E_{\min} < E < 0$ with increasing eccentricity as $E \to 0^-$.

11.2 কক্ষপথের শ্রেণীবিভাগ — Classification of Orbits

Energy	Type	Orbit shape	Bound?
$E = E_{\min}$	Minimum	Circle	Bound
$E_{\min} < E < 0$	Negative	Ellipse	Bound
$E = 0$	Zero	Parabola	Marginally unbound
$E > 0$	Positive	Hyperbola	Unbound

- $E < 0$: **bound orbit** (circle or ellipse).
- $E \geq 0$: **unbound orbit** ($E = 0$: parabola; $E > 0$: hyperbola).

- **Closed orbit**: $\Rightarrow$ only ellipses ($E < 0$) and circles ($E = E_{\min}$) are closed.

আদিবেগ v দ্বারা জ্যামিতিক শ্রেণীবিভাগ — Classification by initial speed v