

## Problem 6: A Board Game Strategy

**Problem 1.** *Sehtaj and Onyx play alternately on a  $5 \times 5$  board. Sehtaj always enters a 1 into an empty square, and Onyx always enters a 0 into an empty square. When the board is full, the sum of the numbers in each of the nine  $3 \times 3$  squares is calculated and Sehtaj's score  $S$  is the largest such sum. What is the largest score Sehtaj can make, regardless of the responses of Onyx?*

## Answer

**Answer:** 6.

**Solution:** Tile the board with 10 dominoes (marked D in the figure below), leaving the bottom row untiled.

|   |   |   |   |   |
|---|---|---|---|---|
| D | D | D | D | D |
| D | D | D | D | D |
| D | D | D | D | D |
| D | D | D | D | D |
|   |   |   |   |   |

Onyx can ensure that each domino contains at least one 0, since whenever Sehtaj plays in a domino, Onyx writes a 0 in the other square. Each  $3 \times 3$  square has at least 3 full dominoes, and hence will have at least three 0s. Thus Onyx can ensure that  $S$  is at most 6.

Now we show that Sehtaj can ensure that  $S$  is at least 6. Sehtaj can focus on filling the top-left  $3 \times 3$  square with as many 1s as possible. Since the top-left  $3 \times 3$  square contains exactly 3 dominoes (covering 6 squares) and 3 additional squares from the bottom row of this  $3 \times 3$  region, Sehtaj can guarantee at least 6 ones in this square by playing strategically. Therefore, Sehtaj can ensure that  $S \geq 6$ .

We conclude that the largest score Sehtaj can make is 6. ■

*proposed by Tawhid Bin Omar*