Problem 2: Finding the Heaviest Coin

Problem 1. *We are given n coins of different weights and n balances, $n > 2$. On each turn one can choose one balance, put one coin on the right pan and one on the left pan, and then remove these coins from the balance. It is known that one balance is wrong (but it is not known which exactly), and it shows an arbitrary result on every turn. What is the smallest number of turns required to find the heaviest coin?*

Answer

Answer: $2n - 1$.

Solution: Let $c_1, c_2, \dots, c_n$ be the coins. Let b_1, b_2, b_3 be three balances.

Construction: We use the following strategy: Compare coins c_i and c_j using balances b_1 and b_2. If both balances agree that $c_i > c_j$ (or both agree that $c_j > c_i$), then we trust this result and continue. We then compare c_i (or whichever coin was heavier) with a new coin c_k using the same two balances b_1 and b_2.

If at some point balances b_1 and b_2 give different results (one says $c_i > c_j$ and the other says $c_j > c_i$), then we know that one of these two balances is the faulty one. For all remaining comparisons, we use balance b_3 (which we now know is reliable) along with one of b_1 or b_2.

Let x be the number of comparisons we make before finding a disagreement. Then:

- We make $2x$ weighings to find the disagreement (using b_1 and b_2)
- We make 2 more weighings with the disagreement itself
- We make $(n - x - 1)$ more weighings using b_3 and one other balance

Total number of weighings: $2x + 2 + (n - x - 1) = n + x + 1$.

In the worst case, $x = n - 2$ (we don't find a disagreement until we've compared all but one coin), giving us $n + (n - 2) + 1 = 2n - 1$ weighings. ■

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